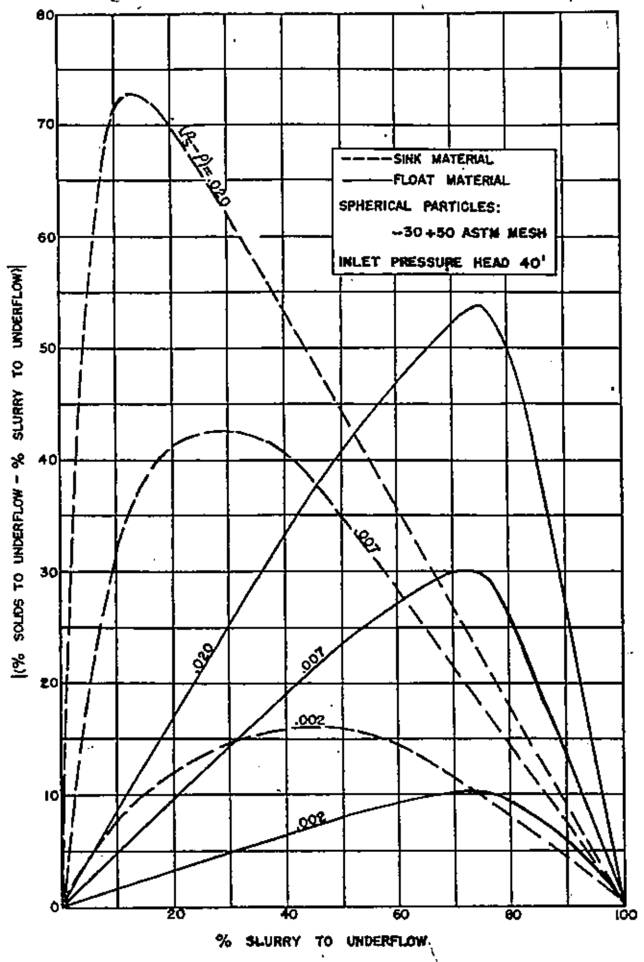
TABLE 13

MULTIPLE CYCLONE TESTS

	Test No. 1		Test No. 2	
	Calculated	Experimental	Calculated	Experimental
5" Inlet Diameter - Inches	0.550	0.550	0.550	0.550
5" Overflow Diameter - Inches	0.884	0.844	0.844	0.844
5" G.P.M.	17.5	17.6	13.4	13.4
4" Inlet Diameter - Inches	0.375	0.375	0.375	0.375
4" Overflow Diameter - Inches	0.550	0.550	0.550	0.550
4" G.P.M.	9.4	10.3	5.8	5.8
3" Inlet Diameter - Inches	0.375	0.375	0.375	0.375
3" Overflow Diameter - Inches	0.364	0.364	0.500	0.500
3" G.P.M.	8.1	7.3	7.6	7.6
Liquid Specific Gravity	1.235	1.235	1.238	1.238
% of Sink Solids to 5" UF	97.5	90.8	96.9	91.1
% of Float Solids to 5" OF	90.0	79.3	95.8	93.3
% of Sink Solids to 4" UF	96.0	61.0	95.8	73.8
% of Float Solids to 4" OF	96.8	91.7	97.3	91.7
% of Sink Solids to 3" UF	97.6	63.9	97.6	89.7
% of Float Solids to 3" OF	89.3	90.4	88.8	87.8
Sink Product Purity %	99.9	99.6	99.9	99.8
Recovery of Sink Material %	99.9	95.2	99.9	98.6
5" Feed Solids Concentration	3.01	3.01	3.00	2.93
5" OF Solids Concentration	1.14	1.28	1.20	1.35
5" UF Solids Concentration	4.33	4.24	4.90	4.99
4" OF Solids Concentration	-	2.33	-	1.98
4" UF Solids Concentration	-	9.54	-	11.75
3" OF Solids Concentration	-	2.11	-	2.79
3" UF Solids Concentration	-	0.73	-	0.66



▲ Fig. 11. Absolute difference between % solids and % slurry to underflow as a function of % slurry to underflow.

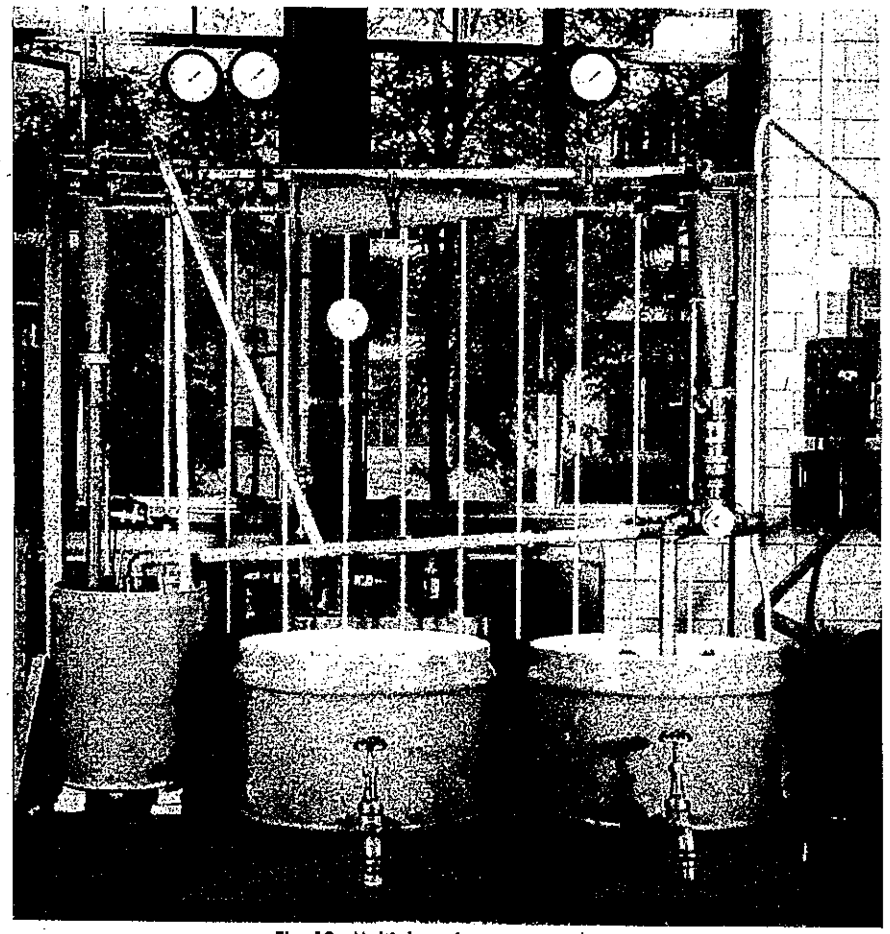


Fig. 12. Multiple cyclone arrangement.

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FLUID FLOW THROUGH PACKED COLUMNS

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The existing information on the flow of fluids through beds of granular solids has been critically reviewed. It has been found that pressure losses are caused by simultaneous kinetic and viscous energy losses, and that the following comprehensive equation is applicable to all types of flow.

$$\frac{\Delta P}{L} g_c = 150 \frac{(1-\epsilon)^2}{\epsilon^3} \frac{\mu U_m}{D_p^{\,2}} + 1.75 \frac{1-\epsilon}{\epsilon^3} \frac{G U_m}{D_p}$$

The equation has been examined from the point of view of its dependence upon flow rate, properties of the fluids, and fractional void volume, orientation, size, shape, and surface of the granular solids. Whenever possible, conditions were chosen so that the effect of one variable at a time could be considered. A transformation of the general equation indicates that the Blake-type friction factor has the following form:

$$f_k = 1.75 + 150 \frac{1-\epsilon}{N_{Re}}$$

A new concept of friction factor, f_v, representing the ratio of pressure drop to the viscous energy term is discussed. Experimental results obtained for the purpose of testing the validity of the equation are reported. Numerous other data taken from the literature have been included in the discussions.

THE pressure loss accompanying the flow of fluids through columns packed with granular material has been the subject of theoretical analysis and experimental investigation. The purpose of the present paper is to summarize the existing information, to verify further experimentally a theoretical development presented earlier, and to discuss practical applications of this new approach. The experimental studies have been confined to gas flow through crushed porous solids. This case is the one usually encountered in practice, but is not identical with the case most thoroughly studied by previous investigators, viz., the flow of fluid through beds of nonporous solids, and more particularly, through solids having uniform geometric shapes.

Factors determining the energy loss (pressure drop) in the packed beds are numerous and some of them are not susceptible to complete and exact mathematical analysis. Various workers in the field have made simplifying assumptions or analogies so that they could utilize some of the general equations representing the forces exerted by the fluids in motion (molecular, viscous, kinetic, static, etc.) to arrive at a useful expression correlating these factors. A survey of the literature reveals various expressions derived from different assumptions, correlating the particular experimental data obtained with or without some of the data published earlier. These correlations differ in many respects; some are to be used only at low fluid flow rates, while others are applicable only at higher rates. A separate survey of all these various correlations is not included here.

As most authorities agree, the factors to be considered are: (1) rate of fluid flow, (2) viscosity and density of the fluid, (3) closeness and orientation of packing, and (4) size, shape, and surface of the particles. The first two variables concern the fluid, while the last two the solids.

1. **Rate of Fluid Flow.** It is known that pressure drop through a granular bed is proportional to the fluid velocity at low flow rates, and approximately to the square of the velocity at high rates. Osborne Reynolds (*23*) was first to formulate the resistance offered by friction to the motion of the fluid as the sum of two terms, proportional respectively to the first power of the fluid velocity and to the product of the density of the fluid with second power of its velocity:

$$\Delta P/L = aU + b\rho U^2 \qquad (1)$$

where ΔP is the pressure loss along the length L, ρ the density of the fluid, U its linear velocity, and a and b are factors which are functions of the system. A transformation of Equation (1) which yields a linear expression is:

$$\Delta P/LU = a + bG \qquad (2)$$

where ρU has been replaced by G, the mass flow rate. The above two-term pressure-drop equation has been found to be satisfactory over the range of flow rates encountered in packed columns. Lindquist (*19*), Morcom (*20*), and Ergun and Orning (*7*) have plotted $\Delta P/LU$ against G and obtained straight lines as expected from Equation (2). The former two authors have included in their plots factors which pertain to the properties of the system. These factors are important and will be discussed later, but they are irrelevant for the purpose of testing the linearity of Equation (2). As a typical plot, data obtained for gas flow through a bed of crushed porous solids are shown in Figure 1. The experimental results of the present investigation and those mentioned above (*7, 19, 20*), as well as the data obtained from the literature (*3, 22*), indicate that the two-term equation accurately expresses the relation between flow rate and pressure drop.

2. **Viscosity and Density of Fluid.** From Equation (2) it is seen that as the velocity approaches zero as a limit, the ratio of pressure drop to velocity will become constant:

$$\lim_{U \to 0} \frac{\Delta P/L}{U} = a \qquad (3)$$

which is a condition for viscous flow. According to the Poiseuille equation and Darcy's law, the factor a is proportional to the viscosity of the fluid. The other limiting condition is reached at high flow rates when the constant a is negligible in comparison to bG. This is a condition for completely turbulent flow where kinetic energy losses constitute the whole resistance. The effect of density is already contained in G. Equation (2) can be rewritten:

$$\Delta P/L = a'\mu U + b\rho U^2 \qquad (4)$$

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where μ is the viscosity of the fluid and a' ($= a/\mu$) is a factor pertaining to the variables of the solids only. The first term of Equation (4) represents viscous energy losses and the second term the kinetic energy losses. The Kozeny equation (*14*) makes use of the viscous energy term to represent the pressure drop, while the Blake (*2*), Burke and Plummer (*3*), and Chilton and Colburn (*5*) approach employs the kinetic energy term and compensates the effect of viscous energy losses with a variable friction factor.

3. **Closeness (Fractional Void Volume) and Orientation of Packing.** Fractional void volume has been one of the most controversial factors in packed systems. Early theoretical treatments were not successful in establishing the dependency of the pressure drop upon fractional void volume. It was Blake who first successfully treated the problem by an approach analogous to that of Stanton and Pannell (*25*) to pressure drop in circular pipes. Blake obtained the following dimensionless groups:

$$\frac{\Delta P g_c}{\rho U^2}\frac{D_p}{L}\frac{\epsilon^3}{1-\epsilon} \text{ and } \frac{D_p \rho U}{\mu(1-\epsilon)}$$

where ϵ is the fractional void volume, g_c the gravitational constant, and D_p the diameter of the solid particles. The first of these groups is recognized as the modified friction factor and the second as the modified Reynolds number. Blake suggested that the former of these groups be plotted against the latter. Since both dimensionless groups contain the fractional void volume, it can be deduced that pressure drop is not a function of a single group alone.

The failure of the earlier attempts to arrive at a useful expression can be attributed to the want of recognition of the fact that pressure drop is caused by simultaneous kinetic and viscous energy losses.

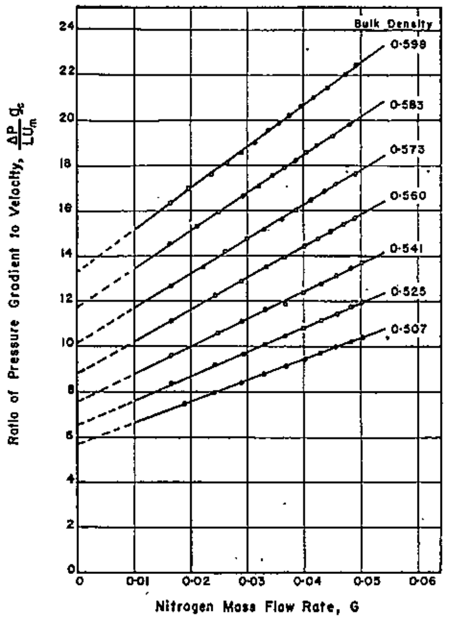

Fig. 1. Typical plots of the linear form of pressure-drop equation for a system packed to different fractional void volumes, Equation (2). Nitrogen flow through 16-20 mesh high temperature oven coke. Particle density = 1.06 g./cc. Cross-sectional area of tube = 7.24 sq.cm. Exit at 724 mm. Hg. and 21° C.

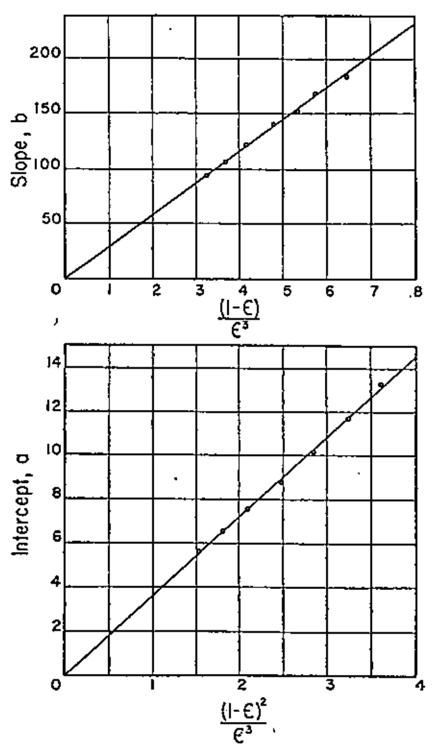

Fig. 2. Dependence of viscous and kinetic energy losses on fractional void volume, Equations (7) and (8). Intercepts and slopes are obtained from data of Figure 1 by method of least squares.

Theoretical considerations of later workers (*3, 7*) indicate that dependency of each energy loss upon fractional void volume is different. Burke and Plummer proposed the theory that the total resistance of the packed bed can be treated as the sum of the separate resistances of the individual particles in it. Accordingly, viscous energy loss was found to be proportional to $(1-\epsilon)/\epsilon^2$ and kinetic loss to $(1-\epsilon)/\epsilon^3$. The authors, however, failed to recognize the additive nature of these losses and correlated the pressure drop by the use of dimensionless groups similar to those of Blake. For viscous flow, Kozeny (*14*) arrived at an equation widely used later (*4, 10, 11, 13, 15, 26*) by assuming that the granular bed is equivalent to a group of similar parallel channels. The derived dependency upon fractional void volume was $(1-\epsilon)^2/\epsilon^3$. This factor is different by a fraction $(1-\epsilon)/\epsilon$ from the factor derived by Burke for viscous flow. Fair and Hatch (*10*), Carman (*4*), Lea and Nurse (*15*), Fowler and Hertel (*11*), and others (*6, 13, 17, 26*) verified the Kozeny factor experimentally. For a general correlation valid at all flow rates, however, Carman recommended the plot of the dimensionless groups of Blake. Recently, Leva (*18*) and Morse (*21*) also adopted Blake's procedure in presenting the pressure drop data in fixed beds. Leva, et al. (*18*) stated that the pressure drop was proportional to $(1-\epsilon)^2/\epsilon^3$ at lower flow rates and to $(1-\epsilon)/\epsilon^3$ at higher flow rates.

Carman noted that at low fluid-flow rates the method of Blake leads to the Kozeny equation, hence to the factor $(1-\epsilon)^2/\epsilon^3$:

$$\frac{\Delta P g_c}{L} = k_1 \frac{(1-\epsilon)^2}{\epsilon^3}\frac{\mu U}{D_p^2} \tag{5}$$

On the other hand, at high flow rates Blake's method gives rise to the equation of Burke and Plummer for turbulent flow:

$$\frac{\Delta P g_c}{L} = k_2 \frac{1-\epsilon}{\epsilon^3}\frac{\rho U^2}{D_p} \tag{6}$$

the factor involving the fractional void volume being $(1-\epsilon)/\epsilon^3$. This range of the plot of Blake has generally been overlooked.

Based on the theory of Reynolds for resistance to fluid flow and the method of Kozeny, a general equation was developed by Ergun and Orning for pressure drop through fixed beds. In summary the following conclusions can be drawn from their work:

1. Total energy losses in fixed beds can be treated as the sum of viscous and kinetic energy losses.

2. Viscous energy losses are proportional to $(1-\epsilon)^2/\epsilon^3$ and the kinetic energy losses to $(1-\epsilon)/\epsilon^3$. Since a and b of Equation (4) represent the coefficients of viscous and kinetic energy losses, respectively, it is expected that a be proportional to $(1-\epsilon)^2/\epsilon^3$ and b to $(1-\epsilon)/\epsilon^3$ in order for the theory to be valid. Although the above authors have correlated their data successfully, single systems have not been thoroughly examined at various fractional void volumes. One of the aims of the present work has been to investigate the single systems at various packing densities. A known amount of solids was packed 6 to 20 different bulk densities each resulting in a different fractional void volume. For each packing the coefficients a and b of Equation (2) were determined from pressure drop and flow rate measurements (Fig. 1). Figures 2 and 3 show typical plots of a against $(1-\epsilon)^2/\epsilon^3$ and b against $(1-\epsilon)/\epsilon^3$ obtained from Figure 1. Such plots yield straight lines each passing through the origin. The graphical representation is simple, yet most effective in the investigation of the function of fractional void volume. A similar procedure has been adopted recently by Arthur, et al. (*1*) in testing the validity of the Kozeny equation and by Ergun (*8*) in connection with particle density determinations for porous solids. It is of interest also to note that the two extreme ranges of the Blake plot lead to the terms of the general equation proposed by Ergun and Orning. The proportionalities can be expressed in the formulae:

$$a = a'' \frac{(1-\epsilon)^2}{\epsilon^3} \tag{7}$$

$$b = b'' \frac{1-\epsilon}{\epsilon^3} \tag{8}$$

where a'' and b'' are factors of proportion-

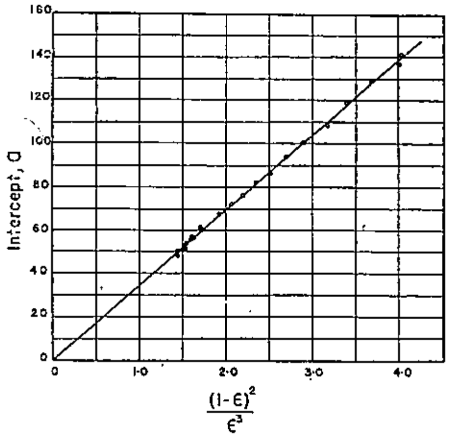

Fig. 3. Dependence of viscous energy losses on fractional void volume, Equation (7). Data obtained for nitrogen flow through 50-60 mesh Eagle coke. Particle density = 1.27 g./cc. Cross-sectional area of the tube = 7.24 sq.cm. Exit gas at 740 mm. Hg. and 23° C.

ality. Their substitution into Equation (2) yields:

$$\frac{\Delta P}{L} = a'' \frac{(1-\epsilon)^2}{\epsilon^3} U + b'' \frac{1-\epsilon}{\epsilon^3}\rho U^2 \tag{9}$$

A rearrangement of Equation (9) leads to:

$$\frac{\Delta P}{LU}\frac{\epsilon^3}{(1-\epsilon)^2} = a'' + b'' \frac{G}{1-\epsilon} \tag{10}$$

Equation (10) makes it possible to group all data of Figure 1 on a single line by plotting

$$\frac{\Delta P}{LU}\frac{\epsilon^3}{(1-\epsilon)^2}$$

against $G/(1-\epsilon)$. This is demonstrated in Figure 4.

Up to this point the aim has been to formulate the effect of fractional void volume in fixed beds, and the effect of orientation was not included. The orientation of the randomly packed beds is not susceptible to exact mathematical formulation. This is especially true if the particles have odd shapes and are not negligible in size compared with the diameter of the container. Furnas (*12*) has treated the subject at length and introduced the concept of "normal packing" which was obtained by a standard procedure. In the present investigation, however, such a concept had to be abandoned. The problem was to pack a known amount of solids to various bulk densities; yet each packing had to be uniform and reproducible.

This was accomplished by admitting gas below the supporting grid after the solids were poured in. The gas rate was sufficient to keep the bed in an expanded state and the use of a vibrator attached to the tube assured the uniformity of the packing. By varying the rate of upward gas flow, the bulk density could be varied from the tightest possible to the loosest stable packing. For crushed material the most tightly packed bed having a height of 30 cm. could easily be expanded by 6 to 7 cm. When the desired packing density was attained, the vibrator was disconnected and the gas flow cut off. The bed then was ready for pressure drop and flow rate measurements. Highly reproducible packings can be obtained by this method, and more important, the particles are believed to be oriented by the gas flowing upward. This is evidenced by the existence of a theoretical relationship (*7*), verified experimentally, between the bed expansion and the flow rate. A further evidence for particle orientation was found in the fact that the most tightly packed beds have been obtained by slowly reducing the rate of upward gas flow to an initially expanded bed while subjecting it to vibration.

It will be evident on inspection of the form of Equation (9) that the estimation of fractional void volume is important, particularly since it enters to second- and third-power terms and is in many cases difficult to measure directly. Whenever the particle density and the total weight of the granular material filling a given volume are known, ϵ may be readily calculated. But the particle density of crushed porous materials is not readily known and its determination has presented a problem which was much discussed. Fractional void volumes were usually calculated by the use of apparent specific gravities which were determined by various procedures. Use of such values for ϵ in the pressure-drop equations necessitated the introduction of correction factors. This often caused the workers to doubt the validity of the factors describing the dependence of pressure drop upon ϵ and to seek new correlations. However, this was believed to be unwarranted (*8*) since the determination of pressure drop through beds of porous particles hinges upon the evaluation of the particle density. Therefore, a gas flow method was developed (*8*) for the determination of the particle density of porous granules. The method was checked by the densities obtained for nonporous solids and the agreement was good. Use of the particle densities of coke obtained by the method described, in the determination of fractional void volume and hence in the pressure drop equation, resulted in excellent agreements.

4. **Size, Shape and Surface of the Particles.** The effect of the particle size and shape is best analyzed in the light of theoretical implications of the Blake plot. The identity between the two extreme ranges of the Blake plot and the theoretical equations developed respectively by Kozeny and Burke for viscous- and turbulent-flow ranges has already been shown. Also, it has been pointed out that these two expressions constituted the following general equation developed by Ergun and Orning (*7*):

$$\Delta P g_c/L = 2\alpha\mu S_v^2 U_m (1-\epsilon)^2/\epsilon^3 + (\beta/8) G U_m S_v (1-\epsilon)/\epsilon^3 \tag{11}$$

where α and β are statistical constants, g_c is the gravitational constant, and S_v is the specific surface of solids, i.e., surface of the solids per unit volume of the solids. Instead of specific surface, S_v, surface per unit packed volume, S, has been employed by some workers. Since the latter quantity involves the fractional void volume, use of specific surface has been preferred in the present work. The relation between the two quantities is expressed by

$$S = (1-\epsilon) S_v.$$

Equation (11) involves the concept of "mean hydraulic radius" in its theoretical development (*7*). Its validity has been tested with spheres, cylinders, tablets, nodules, round sand and crushed materials (glass, coke, coal, etc.) and found to be satisfactory. The experiments have not been extended to include solids having holes and other special shapes. For those

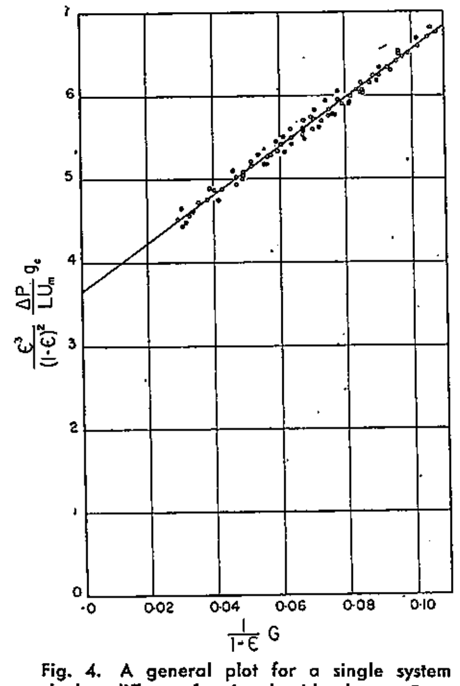

Fig. 4. A general plot for a single system packed to different fractional void volumes. Data of Figure 1 are grouped on a single straight line representing Equation (10).

cases the concept of specific surface was believed to be not applicable by Burke who suggested compensation by empirical factors in connection with the use of the Blake plot.

Determination of specific surface involves the measurement of the solid surface area as well as that of solid volume and presents no problem for uniform geometric shapes. For irregular solids, especially for porous materials, however, surface area determination becomes involved. The surface of porous materials is necessarily full of holes and projections. Different surface areas are usually defined in connection with porous materials, viz., total surface area (including that of pores), external visible surface area, geometric surface area, etc. A geometric surface, as distinct from external visible surface, may be visualized as the surface of an impervious envelope surrounding the body in an aerodynamic sense. Irregularities and striae on the surface would not be taken into full account in a geometric surface area in contrast to external surface area. Whether the value of the total, external or geometric surface area is desired will depend on the purpose for which it is to be used. Geometric surface area is believed (*9*) to be the relevant one in connection with the pressure drop in packed columns. This is made evident by the close agreement between the surface areas determined by gas-flow methods and those by microscopic and light extinction methods. Inasmuch as the surface roughness affects both the geometric surface area and the particle density, the determination of its influence upon pressure drop lies in the evaluation of the effective values of these quantities.

It has been customary to use a characteristic dimension to represent the particle size in pressure-drop calculations. The characteristic dimension generally used is the diameter of a sphere having the specific surface, S_v, which is expressed by

$$D_p = \frac{6}{S_v}$$

Substitution of D_p into Equation (11) yields:

$$\frac{\Delta P g_c}{L} = k_1 \frac{(1-\epsilon)^2}{\epsilon^3}\frac{\mu U_m}{D_p^2} + k_2 \frac{1-\epsilon}{\epsilon^3}\frac{G U_m}{D_p} \tag{12}$$

where $k_1 = 72\,\alpha$ and $k_2 = 3/4\,\beta$. A linear form of Equation (12) is:

$$\frac{\Delta P g_c}{L}\frac{D_p^2}{\mu U_m}\frac{\epsilon^3}{(1-\epsilon)^2} = k_1 + k_2 \frac{N_{Re}}{1-\epsilon} \tag{13}$$

where

$$N_{Re} = \frac{D_p G}{\mu}$$

The left-hand side of Equation (13) is the ratio of pressure drop to the viscous energy term and will be designated by f_v.

$$\frac{\Delta P g_c}{L}\frac{D_p^2}{\mu U_m}\frac{\epsilon^3}{(1-\epsilon)^2} = f_v \tag{13a}$$

$$f_v = k_1 + k_2 \frac{N_{Re}}{1-\epsilon} \tag{13b}$$

According to Equation (13) a linear relationship exists between f_v and $N_{Re}/1-\epsilon$. Data of the present investigation and those presented earlier have been treated accordingly, and the coefficients k_1 and k_2 have been determined by the method of least squares. The values obtained are $k_1 = 150$ and $k_2 = 1.75$ representing 640 experiments. Data involved various-sized spheres, sand, pulverized coke, and the following gases: CO_2, N_2, CH_4 and H_2. Once the constants k_1 and k_2 were obtained it was

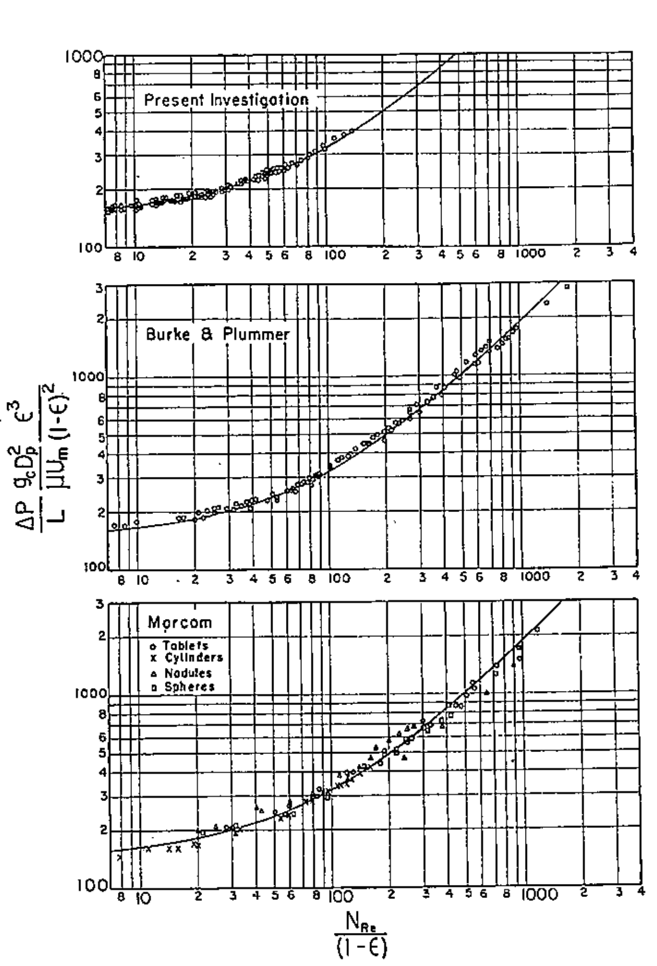

Fig. 5. A general graphical representation of pressure drop applicable to both viscous and turbulent flows for systems considered. Solid lines in all three cases are drawn according to Equation (13c) and are linear on arithmetic scale. The ordinate is represented by f_v Equation (13a).

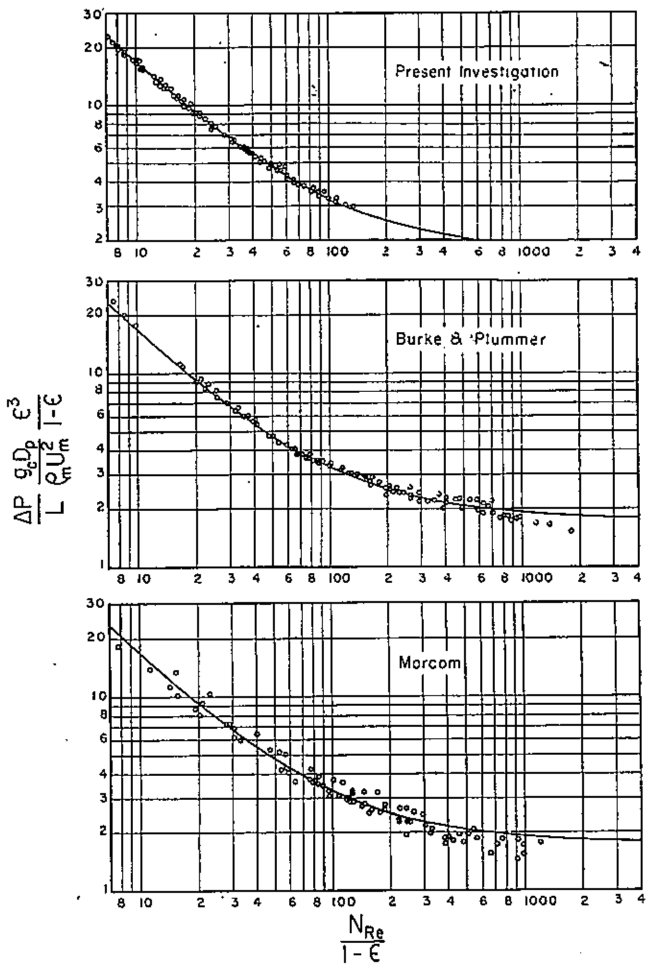

Fig. 7. Graphical representation of pressure drop in fixed beds. Data of Figure 5 are replotted. In all three cases solid lines are identical and are drawn according to Equation (14b). The ordinate is represented by f_k, Equation (14a).

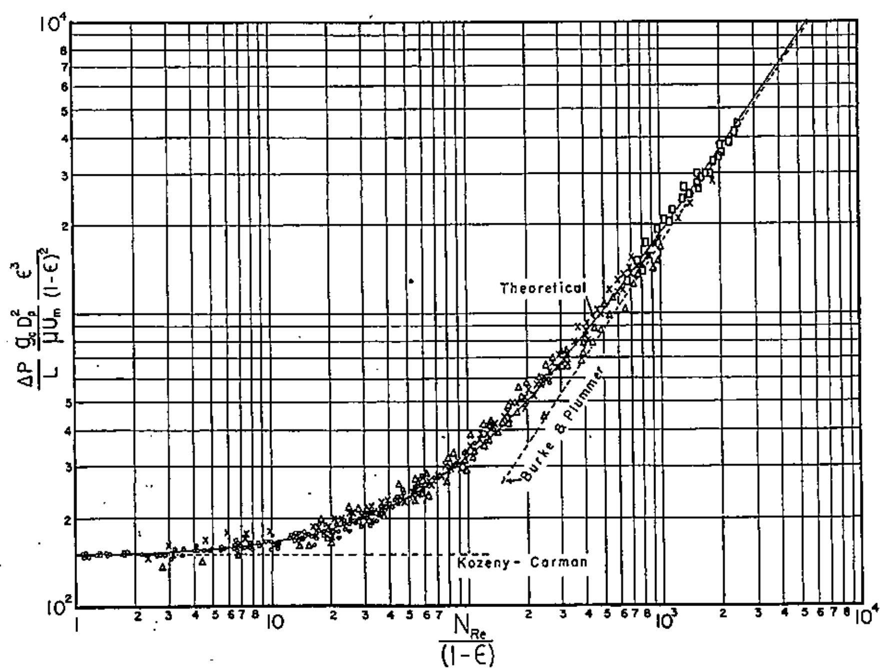

Fig. 6. A comprehensive graphical representation of pressure drop in packed columns. Solid line represents linear Equation (13c).

○ Present investigation
△ Morcom
X Burke and Plummer
□ Oman and Watson.

possible to construct the general equation. The results are shown on the top of Figure 5. To be able to include a wider range of data, a logarithmic scale has been used which results in a curve for the straight line of Equation (13). Data of Burke and Plummer and those of Morcom are also shown in Figure 5. In all three cases the solid lines are identical and are drawn according to the following equation:

$$f_v = 150 + 1.75\frac{N_{Re}}{1-\epsilon} \qquad (13c)$$

Data shown in Figure 5 and some additional data obtained from the literature covering wider ranges of flow rate are included in Figure 6, together with the asymptotes of the resulting curve on the logarithmic scale. Again the solid line represents Equation (13c).

A different form of Equation (12) is represented by:

$$\frac{\Delta P g_c}{L}\frac{D_p}{GU_m}\frac{\epsilon^3}{1-\epsilon} = k_1\frac{1-\epsilon}{N_{Re}} + k_2 \qquad (14)$$

The left-hand side of Equation (14) is the ratio of total energy losses to the term representing kinetic energy losses and will be designated by f_k

$$\frac{\Delta P g_c}{L}\frac{D_p}{GU_m}\frac{\epsilon^3}{1-\epsilon} = f_k \qquad (14a)$$

$$f_k = 150\frac{1-\epsilon}{N_{Re}} + 1.75 \qquad (14b)$$

f_k is similar to the friction factor more commonly used and is identical with the dimensionless group of Blake. It will be noted that Burke and Plummer plotted essentially f_k vs. $(1-\epsilon)/M_{Re}$ which plot, according to Equation (14), should yield a straight line on an arithmetic scale. The authors apparently failed to recognize this fact. The best curve drawn through the experimental points on an arithmetic scale does not differ markedly from the line representing Equation (14b). The scatter to be seen on the plot of Burke and Plummer was largely due to the systems involving mixtures and those for which the ratio of tube diameter to particle size was less than 10. These systems have been omitted in Figure 5. It has been customary, however, to plot f_k against $N_{Re}/(1-\epsilon)$ instead of the inverse of the last variable. This type of plot is the one suggested by Blake and adopted by Carman, Morse and others. Figure 7 shows f_k plotted vs. $N_{Re}/(1-\epsilon)$ for the data already presented in Figure 5. Figure 8 is a more comprehensive presentation. The solid lines are drawn according to Equation (14b). A comparison of Figure 6 with 8 is analogous to that of f_v with f_k. Both plots are capable of presenting the data. However, f_v has a big advantage over f_k in that it is a linear function of the modified Reynolds number, viz., $N_{Re}/(1-\epsilon)$. The curve of Figure 6 is a straight line on an arithmetic scale. On the other hand, f_k, which has been used almost exclusively, is an inverse function. A comparison of various empirical representations with Equation (12) is to be seen in Figure 9.

The foregoing treatment so far has been confined to studying the factors involved in the pressure loss in packed beds and to analyzing experimentally the theoretical developments presented earlier. It is only proper that the equations presented are also analyzed briefly from the standpoint of pure fluid dynamics. Fortunately, the equations lend themselves for such analyses. By definition:

$$D_p = 6/S_v \qquad (15a)$$

and

$$S_v = S_t/AL(1-\epsilon) \qquad (15b)$$

where S_t = total geometric surface area of the solids and A = cross-sectional area of the empty column. The total force exerted by the fluid on the solids $= \Delta P g_c A\epsilon$, therefore the tractive force per unit solid surface area, usually referred to as the shear stress, τ, is expressed by:

$$\tau = \Delta P g_c A\epsilon/S_t \qquad (15c)$$

The ratio of the volume occupied by the fluid in the bed, $AL\epsilon$, to the surface area it sweeps, S_t, is the hydraulic radius, r_h,

$$r_h = AL\epsilon/S_t \qquad (15d)$$

The actual average velocity of the fluid in the bed is obtained from the ratio of the superficial fluid velocity to the fractional voids,

$$u = U/\epsilon \qquad (15e)$$

Substitution of Equations (15a-e) into Equation (13a) gives

$$f_v = 36\frac{\tau r_h}{\mu u} \qquad (16)$$

and into Equation (14a) gives

$$f_k = 6\frac{\tau}{\rho u^2} \qquad (17)$$

Similarly proper substitution will yield

$$\frac{N_{Re}}{1-\epsilon} = \frac{6\rho u r_h}{\mu} \qquad (18)$$

Therefore, Equations (13) and (14) respectively will become:

$$36\frac{\tau r_h}{\mu u} = 150 + 1.75\frac{6\rho u r_h}{\mu} \qquad (19)$$

and

$$6\frac{\tau}{\rho u^2} = 150\frac{\mu}{6\rho u r_h} + 1.75 \qquad (20)$$

It is seen that these transformations employing the absolute values of shear stress, fluid density, and velocity eliminate the fractional void volume. The terms involved in Equations (16-20) are well known in the fields of hydro- and aerodynamics. Other forms of dependences upon ϵ ascribed to a general equation, as encountered in the literature, would not lead to complete elimination of the fractional void volume upon transformation to these fundamental variables.

The theoretical significances of the constants k_1 and k_2 have been omitted in the foregoing treatment. The former of these constants is discussed by Carman and Lea and Nurse (*15*) in connection with the Kozeny equation. As a result of comparison of various systems involving different fractional void volumes, Lea and Nurse (*16*) concluded that $a(=k_1/72)$ was not a constant but varied with the fractional void volume. Whether or not k_1 is a constant is to be decided on inspection of the lower end of Figure 6 and the upper end of Figure 8 where viscous energy losses are dominant. However, the inherent inaccuracies involved in the measurements of specific surface, fractional void volume, etc., must be borne in mind. In the present work, moreover, single systems were investigated at different fractional void volumes and no evidence of variance of k_1 with ϵ was found. This point is clearly supported by the proportionality of a to $(1-\epsilon)^2/\epsilon^3$ as to be seen from Figures 2 and 3, and similar other graphical representations (*1, 8, 9*). The factor $k_2(=3/4\beta)$ is subject to treatment similar to that of k_1 (*7, 8, 9*).

Summary

The laws of fluid flow through granular beds have several aspects of practical consequence. They generally find use in correlating the rate of mass and heat transfer to and from moving fluids (*24*). The extension of such relationships to packed columns will require formulation of the laws of fluid flow through granular beds. Empirical correlations are generally useful for the particular purpose for which they are made, but may not shed light for a different purpose. For the sake of clarity in the application and use of the data obtained in packed columns, it seemed desirable to develop expressions (Equation (12)) in a comprehensive form applicable to all types of flow. In doing so the theoretical developments, as well as the empirical approaches, have been considered and the following conclusions have been drawn:

1. Total energy loss in fixed beds can

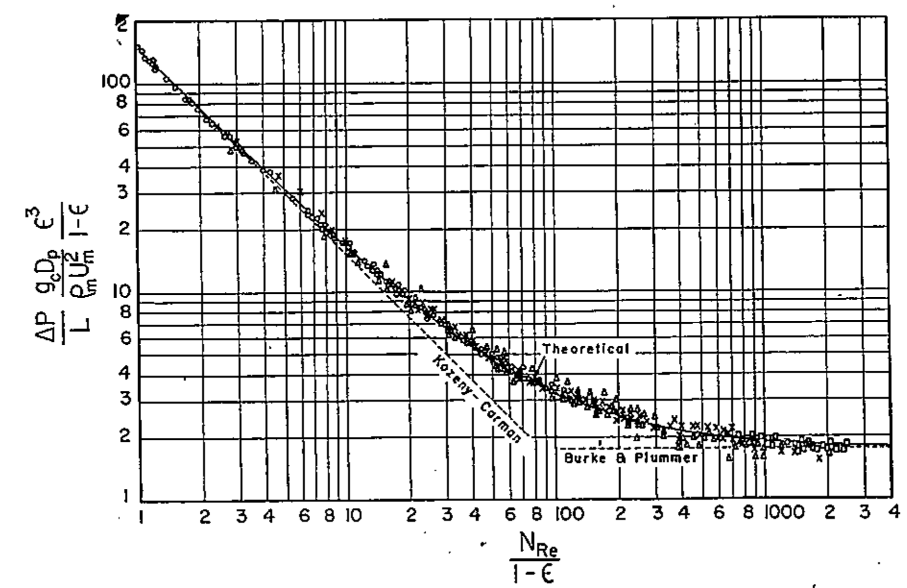

Fig. 8. A comprehensive plot of pressure drop in fixed beds. Data of Figure 6 are replotted. This type of plot is identical with that of Blake. Solid line is drawn according to Equation (14b).

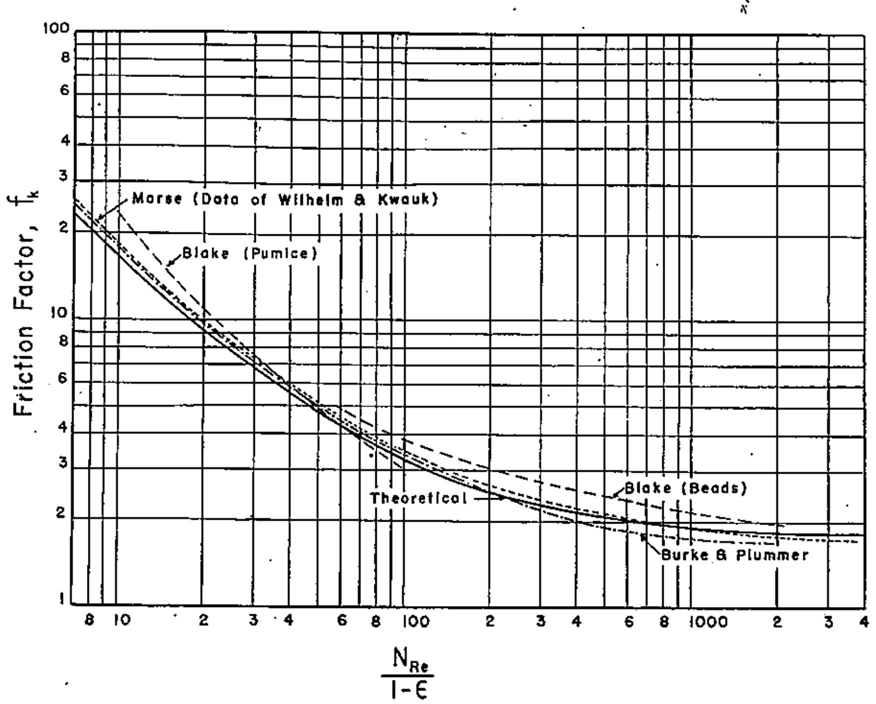

Fig. 9. Comparison of various empirical representations with Equation (12).

be treated as the sum of viscous and kinetic energy losses.

2. Viscous energy losses per unit length are expressed by the first term of Equation (12):

$$150\,\frac{(1-\epsilon)^2}{\epsilon^3}\,\frac{\mu U_m}{D_p{}^2}$$

and the kinetic energy losses by the second term:

$$1.75\,\frac{1-\epsilon}{\epsilon^3}\,\frac{\rho_m U_m{}^2}{D_p}$$

3. For any set of data the relative amounts of viscous and kinetic energy losses can be obtained from either Equation (13) or (14).

4. A new form of friction factor, f_v, representing the ratio of pressure drop to the viscous energy term has been given (Equation 13c) and should have advantages over the conventional type of friction factor.

5. A linear equation has been shown to represent the conventional type of friction factor, viz., the ratio of pressure drop to energy term representing kinetic losses (Equation 14b).

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Notation

a, a', a'' = coefficients in Equations (1), (4), and (7), respectively
A = cross-sectional area of the empty column
b, b'' = coefficients in Equations (1) and (8), respectively
D_p = effective diameter of particles as defined by Equation (15a)
f_v = friction factor, which represents the ratio of pressure loss to viscous energy loss and which is linear with mass flow rate, defined by Equation (13a)
f_k = friction factor, identical with the dimensionless group of Blake, defined by Equation (14a)
g_c = gravitational constant
G = mass-flow rate of fluid, $G = \rho U$
k_1 = coefficient of the viscous energy term in Equation (12); $k_1 = 150$
k_2 = coefficient of the kinetic energy term in Equation (12); $k_2 = 1.75$
L = height of bed
N_{Re} = Reynolds number, $N_{Re} = D_p G/\mu$
P = pressure loss, force units
r_h = hydraulic radius of packed bed, defined by Equation (15d)
S = surface of solids per unit volume of the bed
S_t = total surface area of the solids in the bed
S_v = specific surface, surface of solids per unit volume of solids
u = actual velocity of fluid in the bed
U = superficial fluid velocity based on empty column cross section
U_m = superficial fluid velocity measured at average pressure
α = coefficient of viscous energy term in Equation (11)
β = coefficient of kinetic energy term in Equation (11)
ϵ = fractional void volume in bed
μ = absolute viscosity of fluid
ρ = density of fluid
τ = average shear stress, defined by Equation (15c)

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APPLICATIONS OF FLUID MECHANICS AND SIMILITUDE TO SCALE-UP PROBLEMS

PART II

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A brief review of the principles of dynamic similitude is given. Basic concepts of fluid mechanics are used to develop relations between fluid motion, equipment size, and fluid properties that may apply to chemical engineering work. A general method is given whereby the requirements for dynamic similitude for any flowing system can be determined. Examples are given showing how these principles can be used in pilot plant models to obtain scale-up data for operations involving resistance to fluid flow, discharge of liquids from tanks, blending of liquids by a mixer, control of forced vortexes, dissolving of solids during mixing, and absorption and desorption of gases in moving liquids. Suggestions are made for applying the principles to any type of operation involving mass transfer in a liquid, and to other flow operations such as fluidized systems and suspensions.

Applications

Pilot plant operations may be thought of as of two types; those involving the kinematic properties such as viscosity and weight, and those which in addition to the kinematic properties are affected by vapor pressure, solubility, equilibrium, conductivity, and the like. The first type deals with fluid mechanics only, and can be illustrated by examples of fluid flow and certain mixing operations. The second type deals also with chemical reactions and concentrations, and includes such operations as gas absorption and extraction.

Typically Fluid Mechanical. The resistance of flow of fluids through equipment such as pipe lines and heat exchanges can be determined by model studies. The familiar Fanning friction factor and other similar relations are of common knowledge. In these systems it has been shown that the inertia group and the Reynolds number are sufficient to define dynamic similarity when all linear ratios are constant. Therefore, the resistance to flow measured in a model can be projected to the resistance in any larger geometrically similar system, and pumps and pipelines can be sized with accuracy. In such systems where there is no free liquid surface and where the fluid is continuous and incompressible, the Froude, Weber, and Cauchy parameters do not apply, and the Reynolds number and the inertia group alone control. A liquid of any viscosity and density can be used in the model to determine the relationship of the inertia group to the Reynolds number, and the results will apply to any other liquid in scale-up by using Equations (1)-(12).

The blending of miscible liquids is a mixing operation often carried out under conditions represented by curve *ABCD* of Figure 1. Normally the operations are carried out under conditions equivalent to the *CD* part of the curve, using side-entering propellers, properly off-centered, or other types of impellers using baffles. Viscous forces do not exert an influence. Under these conditions there will not be a vortex in the surface of the liquid, and weight or gravity forces will not play a part. A pilot plant model can, therefore, be constructed, and the time and power necessary to achieve a desired blend for a given set of feed conditions can be found. These conditions can be projected to a large size tank by keeping the inertia group constant, geometric similarity and equal liquid flow velocities.

The following example is taken from actual operating data in small and large tanks: A 6-in. diam. square pitch three-blade marine-type propeller operating in a side-entering properly off-centered position in a 15-ft. diam. vertical axis cylindrical tank, was found to blend two gasolines in 40 min. when running at a speed of 1680 rev./min. What size mixer and power are required to blend the two gasolines in the same length time in a 60-ft. diam. tank?

The scale-up will be made on the basis of equal inertia groups, geometric similarity, and equal flow velocities at corresponding points.

All linear dimensions will increase by the ratio of tank size or 60/15 = 4. Liquid depth will be four times that in the smaller tank: the propeller will be a geometrically similar one 2 ft. in diameter. Liquids are the same. Convert force to power P by $FL/T = P$, then the inertia group may be written $PT^3/\rho L^5$, and

$$\frac{P_1 T_1{}^3}{\rho_1 L_1{}^5} = \frac{P_2 T_2{}^3}{\rho_2 L_2{}^5}$$

$$\frac{P_2}{P_1} = \left(\frac{T_1}{T_2}\right)^3 \left(\frac{L_2}{L_1}\right)^5$$

since time occurrences (and velocities) are to be constant $T_1 = 0.25 T_2$ (see line one, Table 2)

then

$$\frac{P_2}{P_1} = (0.25)^3 (4)^5$$

or

$$P_2 = 16\ P_1$$

Actually, the power required by the model is 0.768 hp., thus the power required by the 24-in. propeller is 16 (0.768) = 12.5 hp. The rotational

Pt. I of this paper was run in the January issue.