Heat transfer enhancement of nanofluids

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Abstract

This paper presents a procedure for preparing a nanofluid which is a suspension consisting of nanophase powders and a base liquid. By means of the procedure, some sample nanofluids are prepared. Their TEM photographs are given to illustrate the stability and evenness of suspension. The theoretical study of the thermal conductivity of nanofluids is introduced. The hot-wire apparatus is used to measure the thermal conductivity of nanofluids with suspended copper nanophase powders. Some factors such as the volume fraction, dimensions, shapes and properties of the nanoparticles are discussed. A theoretical model is proposed to describe heat transfer performance of the nanofluid flowing in a tube, with accounting for dispersion of solid particles. © 2000 Elsevier Science Inc. All rights reserved.

Keywords: Nanofluid; Enhanced heat transfer; The hot-wire apparatus; Dispersion

1. Introduction

Low thermal conductivity of process fluid hinders high compactness and effectiveness of heat exchangers, although a variety of techniques is applied to enhance heat transfer. Improvement of the thermal properties of energy transmission fluids may become a trick of augmenting heat transfer. An innovative way of improving the thermal conductivities of fluids is to suspend small solid particles in the fluids. Various types of powders such as metallic, non-metallic and polymeric particles can be added into fluids to form slurries. The thermal conductivities of fluids with suspended particles are expected to be higher than that of common fluids. An industrial application test was carried out by Liu et al. (1988) and Ahuja (1975), in which the effect of particle volumetric loading, size, and flow rate on the slurry pressure drop and heat transfer behavior was investigated. In conventional cases, the suspended particles are of μm or even mm dimensions. Such large particles may cause some severe problems such as abrasion and clogging. Therefore, fluids with suspended large particles have little practical application in heat transfer enhancement.

Application of nanoparticles provides an effective way of improving heat transfer characteristics of fluids (Eastman et al., 1997). Particles <100 nm in diameter exhibit properties

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Notation

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<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>$A$</td>
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<td>specific heat</td>
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<td>$h$</td>
<td>convective heat transfer coefficient</td>
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<td>thermal conductivity</td>
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<tr>
<td>$m$</td>
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<td>particle empirical shape factor</td>
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Greek

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<td>$\rho$</td>
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* Corresponding author.
different from those of conventional solids. Compared with micron-sized particles, nanophase powders have much larger relative surface areas and a great potential for heat transfer enhancement. Some researchers tried to suspend nanoparticles into fluids to form high effective heat transfer fluids. Choi (1995) is the first who used the term nanofluids to refer to the fluids with suspended nanoparticles. Some preliminary experimental results (Eastman et al., 1997) showed that increase in thermal conductivity of approximately 60% can be obtained for the nanofluid consisting of water and 5 vol% CuO nanoparticles.

By suspending nanophase particles in heating or cooling fluids, the heat transfer performance of the fluid can be significantly improved. The main reasons may be listed as follows:

1. The suspended nanoparticles increase the surface area and the heat capacity of the fluid.
2. The suspended nanoparticles increase the effective (or apparent) thermal conductivity of the fluid.
3. The interaction and collision among particles, fluid and the flow passage surface are intensified.
4. The mixing fluctuation and turbulence of the fluid are intensified.
5. The dispersion of nanoparticles flattens the transverse temperature gradient of the fluid.

2. Preparation of nanofluids

Preparation of nanofluids is the first key step in applying nanophase particles to changing the heat transfer performance of conventional fluids. The nanofluid does not simply refer to a liquid–solid mixture. Some special requirements are necessary, such as even suspension, stable suspension, durable suspension, low agglomeration of particles, no chemical change of the fluid. In general, these are effective methods used for preparation of suspensions: (1) to change the pH value of suspensions; (2) to use surface activators and/or dispersants; (3) to use ultrasonic vibration. All these techniques aim at changing the surface properties of suspended particles and suppressing formation of particles cluster in order to obtain stable suspensions. It depends upon the application case how these techniques are used.

The common activators and dispersants are thiols, oleic acid, laurate salts. Selection of the suitable activators and dispersants mainly depends upon the properties of solutions and particles. For example, the aqueous-favoring dispersant may be fit for water–particle suspension. In this article, the nanostructured material of Cu particles of about 100 nm diameter is used to form water– and mineral oil–particle suspensions. The nanoparticles and a fluid are directly mixed. While preparing the suspensions, different types and percentages of activators or dispersants have been tried and tested.

Example 1 (Transformer oil–Cu nanoparticles suspension). Cu nanoparticles are mixed with the transformer oil by 2 and 5 vol%, respectively. To stabilize the suspension, oleic acid is selected as the dispersant to cover the nanoparticles. The amount of mixed oleic acid is calculated with weight percentage of Cu particles. Several percentages of oleic acid have been tested. The suspension is vibrated for 10 h in an ultrasonic vibrator. The experimental results show that in the case that the percentage of oleic acid amounts to 22 wt% of the particles, the stabilization of the suspension can last about 1 week in the stationary state and no sediment is found. The distribution and cluster of the ultra-fine copper particles have been examined by a HITACHI H-8 electron microscope. Fig. 1 gives TEM photographs of the suspension of transformer oil–Cu nanoparticles. The electron micrographs show that the particles are dispersed in the fluid and some clustering occurs.

Example 2 (Water–Cu nanoparticles suspension). The suspension contains 5 vol% Cu nanoparticles. The laurate salt is used to enhance stability of the suspension. Several percentages of the laurate salt (2, 4, 6, 8, 9 wt% via the particle) have been tested. The best case corresponds to the percentage of 9 wt%, which means that 9 wt% may be the minimum value for forming a stable water–Cu particle suspension in this case. After the suspension has been vibrated in a ultrasonic vibrator, the stabile suspension can last more than 30 h in the stationary state. Fig. 2 gives TEM photographs of the suspension of water–Cu nanoparticles. Both these micrographs show that the particles are dispersed in deionized water and some clustering occurs.

Comparison between Figs. 1 and 2 and observation of the suspensions reveal that with respect to dispersion behavior and stability, the suspension of Cu particles in transformer oil has superior characteristics to the suspension of Cu particles in

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Fig. 1. TEM micrographs of nanoCu particles–transformer oil at pH = 6.3. (a) 2 vol% suspension (scale times: 100,000). (b) 5 vol% suspension (scale times: 100,000).
water. This explains that the viscosity of fluids may be an important factor affecting the dispersion of ultra-fine particles and the stability of suspensions. The properties of activators and dispersants also play a role in preparing the suspensions.

3. Thermal conductivity of nanofluids

As mentioned before, nanofluids exhibit superior heat transfer characteristics to conventional heat transfer fluids. One of the reasons is that the suspended particles remarkably increase thermal conductivity of nanofluids. The thermal conductivity of nanofluid is strongly dependent on the nanoparticle volume fraction. So far it has been an unsolved problem to develop a sophisticated theory to predict thermal conductivity of nanofluids, although there exists some semi-empirical correlation to calculate the apparent conductivity of nanofluids, although there exists some semi-empirical correlation to calculate the apparent conductivity of nanofluids.

For obtaining a rough estimation, the effective thermal conductivity of solid–liquid mixtures was introduced by Wasp (1977)

$$k_{\text{eff}} = \frac{k_p + 2k_i - 2\pi(k_i - k_p)}{k_p + 2k_i + \pi(k_i - k_p)},$$

(4)

The volume fraction $\alpha$ of the particles is defined as

$$\alpha = \frac{V_p}{V_I + V_p} = \frac{\pi d_p^3}{6 n d_s^3},$$

(5)

where $m$ is the number of the particles per unit volume and $d_s$ is the average diameter of the particles. Comparison between Eqs. (2) and (4) reveals that Wasp’s model is a special case with the sphericity 1.0 of the Hamilton and Crosser’s model.

Both these formulas are applicable for the two-phase mixtures that contain powders with particle diameters on the order of micrometer or even millimeters. In the case of the absence of a suitable formula for predicting the thermal conductivity of nanofluids, the above-mentioned formulas may approximately be applied to obtain a rough estimation.

Applying the Hamilton and Crosser model to a water–alumina nanoparticles suspension, the effective thermal conductivity $k_{\text{eff}}$ is estimated for the values of $\psi$ from 0.3 to 1.0. The effects of particle volume fraction and sphericity on the thermal conductivity are shown in Fig. 3. In addition, the calculated results are compared with the preliminary experimental results (Eastman et al., 1997) in Fig. 3. For a given particle shape, the effective thermal conductivity of suspensions containing solid particles increases with the volume fraction of the solid particles. If the sphericity of alumina nanoparticles is 0.3, a dramatic improvement in the thermal conductivity of nanofluids is expected with a factor 1.2 at the sphericity 0.3, a dramatic improvement in the thermal conductivity of nanofluids is expected with a factor 1.2 at the sphericity 0.3. The dimensions and properties of nanoparticles exert overwhelming effects on the thermal conductivity of suspensions. These results predict that nanoparticles increase the thermal conductivity of conventional heat transfer fluids.
4. Measurement of thermal conductivity of nanofluids

Nanofluids are expected to exhibit superior heat transfer properties compared with conventional heat transfer fluids. One of the reasons is that the suspended particles remarkably increase thermal conductivity of nanofluids. It is known that the thermal conductivity of nanofluid is strongly dependent on the volume fraction dimensions and properties of nanoparticles. It is almost impossible to precisely determine the thermal conductivity of nanofluid by a theoretical approach. In order to determine the thermal conductivity of nanofluids, the experimental approach is needed. Therefore, the transient hot-wire method is used to measure the thermal conductivity of nanofluids.

In the ideal mode of the transient hot-wire apparatus, an infinitely long, vertical, line source of heat possessing zero heat capacity and infinite thermal conductivity is immersed in an infinitely long, vertical, line source of heat possessing zero heat flux. For the line source of radius $r_0$ and a uniform initial temperature, the relationship between the temperature rise of the wire and the thermal conductivity of the fluid was introduced by Wakeham et al. (1991)

$$\Delta T(r_0, t) = \frac{q}{4\pi k} \ln \left( \frac{4at}{r_0^2C} \right). \quad (6)$$

Eq. (6) is the fundamental equation of the transient hot-wire technique. If the slope of the line $\Delta T$ vs $\ln t$ is obtained, the thermal conductivity of the fluid can be determined. This method was also used by Masuda et al. (1993) to measure the thermal conductivity of liquid containing ultrafine particles.

In the transient thermal conductivity measurement apparatus, two platinum wires of 0.153 and 0.063 m length are used, with a wire radius of 50 mm. The two platinum wires are both the line source of heat and a thermometer. The two wires are nominally identical except for their length. They are respectively immersed in two cells of 30 mm diameter. The cells contain sample nanofluids. Both the wires are subject to the same heating current then the same end effects in each is provided. Thus, the difference of the temperature rises of the two wires corresponds to the temperature rise of a finite section of an infinite wire. Therefore, the end effect is eliminated experimentally.

An automatic Wheatstone bridge is used to measure the resistance difference which is related to the difference of the temperature rises of two wires. A HP34401 digital voltmeter is used to measure the difference voltage of the bridge. An experiment lasts about 5 s. If the time is longer, the temperature difference between the hot-wire and the sample fluid increases and free convection takes place, which may result in errors. The temperature rise of the hot wire is a function of time and the heat flux. From Eq. (6) the thermal conductivity of the nanofluids are obtained.

Before measuring the thermal conductivity of nanofluids, the hot-wire apparatus is calibrated by measuring a sample liquid with known thermal conductivity, which shows a high precision of the experimental system. With the apparatus, the thermal conductivities of the transformer oil-copper nanofluid suspension and the water–copper nanoparticles suspension (Cu particles of about 100 nm diameter) are measured. The effects of particle volume fraction on the thermal conductivity for the nanofluids are plotted in Figs. 4 and 5. Figs. 4 and 5 give the effects of particle volume fraction on the thermal conductivity for the transformer oil–copper nanofluid and for the water–copper nanofluid, respectively. In addition, the experimental results are compared to the preliminary experimental results (Eastman et al., 1997).

The results show that one of the factors affecting the thermal conductivity of the nanofluids is the particle volume fraction. The effective thermal conductivity of the nanofluids increases with the volume fraction of the Cu nanoparticles. Comparison with the experimental results and the preliminary experimental results of Eastman et al. (1997) shows that Eastman’s nanofluids exhibit more excellent properties than the sample nanofluids. The primary reason may be that Eastman’s nanofluids use CuO nanoparticles of 36 nm diameter and Cu nanoparticles of 18 nm diameter, but the diameter of Cu nanoparticles suspended in the sample nanofluids varies up to 100 nm. Because the ratio of the surface area to volume for a particle with 20 nm diameter is 5 times larger than that for a particle with 100 nm diameter, a dramatic improvement in effective thermal conductivity of nanofluids is expected by decreasing the particle size. However, the preparation method of Eastman’s nanofluid is expensive and it is difficult to satisfy...
The dispersed model is adopted. This model is often used to handle the movement of the particles in the main flow into account, the phenomena of Brownian diffusion, sedimentation, dispersion, Brownian forces, friction between the fluid and solid particles, and the thermal dispersion coefficient. The slip velocity between the fluid and particles may not be zero, although the particles are ultra-fine. To take the random movement of the particles in the main flow into account, the dispersed model is adopted. This model is often used to handle multi- and one-dimensional complicated diffusion problems.

5. Enhanced heat transfer analysis

Compared to the existing techniques for enhancing heat transfer, the nanofluids show the superior potential of increasing heat transfer rates in a variety of application cases. Choi (1995) quantitatively analyzed some potential benefits of nanofluids for augmenting heat transfer and reducing size, weight and cost of thermal apparatuses, while incurring little or no penalty in pressure drop. There are two different approaches to investigate the enhanced heat transfer of the suspensions: the two-phase one and the single-phase one. The first approach provides the possibility of understanding the functions of both the fluid phase and the solid particle in the heat transfer process, but needs much computation time and computer capacity. By combining Lagrangian statistics and DNS (direct numerical simulation), Sato et al. (1998) applied this approach to analyze the mechanism of two-phase heat and turbulent transport by solid particles (on the micrometer order) suspended in a gas flow, by assuming that the particle enthalpy does not affect the temperature field. The second approach may be considered as a scalar and to be isotropical, i.e., \( D = D_s = D \). Therefore, the apparent thermal conductivity \( k_{app} = k_{eff} + D \). In general, the thermal dispersion coefficient should be determined experimentally, although there exists some analytic correlation of predicting this parameter under a number of assumptions. For a turbulent flow through a tube, for example, Beckman et al. (1990) approximately derived the following formula to calculate the thermal dispersion coefficient:

\[
D = \left\{ \begin{array}{ll}
10.1 Ru \sqrt{f/2} + 5.03 Ru & \text{in cases of large temperature gradients,} \\
[10.1 Ru \sqrt{f/2} + 5.03 Ru]/(1 + l/\sqrt{2f}) & \text{in adiabatic cases,}
\end{array} \right.
\]

(8)

where \( f \) is the Fanning friction factor and \( R \) is the radius of the tube.

It is expected that for the nanofluid flowing through a tube, the thermal dispersion coefficient may have the form:

\[
D = f \left( Re, Pr, \frac{k_d}{k}, \frac{(pc_p)_d}{(pc_p)_l} \right).
\]

(9)

Experiment is necessary to determine this coefficient. If the flow is laminar and fully developed, the velocity \( u \) is given by the Poiseuille–Hagen distribution,

\[
u = 2 \pi \left( 1 - R^2 / R^2 \right),
\]

(10)

where \( \pi \) is the average axial velocity in the x-direction. For the case of plug flow, \( u = \text{const.} \).

If the axial temperature gradient is negligible, Eq. (7) can be simplified as

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r} \left[ r \left( \frac{\partial T}{\partial r} + \frac{D}{(pc_p)_eff} \frac{\partial T}{\partial r} \right) \right].
\]

(11)

According to the given boundary and initial conditions, one is able to find the solutions. For Eq. (11), some boundary conditions are as follows:

\[
T_{i=\infty} = T_u,
\]

(12a)

\[
T |_{r=0} = T_0.
\]

(12b)

Taylor (1953) may have been the first to apply this model to simulating salt diffusion in water. The concept of backmixing is often applied to describe the effect of random movement of fluid superimposed on the main flow (Mecklenburg and Hartland, 1975). The effect of dispersion or backmixing is to flatten the temperature gradient. The contribution of the nanoparticles to convective heat transfer enhancement can be understood from the two aspects: The particles increase the thermal conductivity of fluid and the chaotic movement of the particles strengthens energy transport process. On the basis of the dispersion model, the governing differential equation for the heat transfer process between the nanofluid flowing in a tube and the tube surface yields
By applying the separation principle of variables, the steady-state solution to Eq. (11) can be obtained for the case of plug flow:

\[ \frac{T - T_w}{T_0 - T_w} = 2 \sum_{n=1}^{\infty} e^{-\beta_n^* \pi} \frac{J_0(\beta_n \pi)}{J_1(\beta_n) \beta_n} \]  

(13)

where \( \pi = r/R \), \( \sigma = x/L \), \( Nu = 2Rh/k_{\text{eff}}^* \), \( Pe^* = uL/k_{\text{eff}}^* \), \( Pe^* \) is the modified Pe number, and eigenvalues \( \beta_n^* \)'s are the positive roots of the following equation:

\[ J_0(\beta_n^*) = 0. \]  

(14)

According to the heat transfer relation

\[ -k_{\text{eff}}^* \frac{\partial T}{\partial r} \bigg|_{r=R} = h(T_b - T_w) \]  

(15)

one can derive the analytical correlation of \( Nu \) number (Fig. 6).

\[ Nu = \frac{\sum_{n=1}^{\infty} e^{-\beta_n^* \pi}}{\sum_{n=1}^{\infty} e^{-\beta_n^* \pi}/\beta_n^*}. \]  

(16)

Although this correlation is formally similar to that for a pure fluid flowing in a duct, the local \( Nu \) number (i.e., heat transfer coefficient) depends upon both the thermal conductivity and the thermal dispersion coefficient which indicates effect of irregular movement of nanoparticles in the main flow. If the dispersion in both the axial and radial directions should be taken into account, Eq. (7) is needed. In this case, Dankwerts’ conditions (1953) are adopted to describe the axial boundary conditions. The boundary conditions in the radial direction are modified as follows:

\[ -k_{\text{eff}}^* \frac{\partial T}{\partial r} = u\alpha \rho c_p (T_0 - T) \quad \text{at} \quad r = 0, \]  

(17a)

\[ \frac{\partial T}{\partial r} = 0 \quad \text{at} \quad r = R. \]  

(17b)

If the boundary conditions in the radial direction remain the same, the solution to Eq. (7) is derived

\[ \frac{T - T_w}{T_0 - T_w} = \frac{2}{\sum_{n=1}^{\infty} \frac{X(\pi)}{X(0) - X'(0)/Pe^*}} \frac{J_0(\beta_n \pi)}{J_1(\beta_n) \beta_n}. \]  

(18)

where

\[ X(\pi) = m_1 e^{\pi^2} - m_2 e^{\pi_1^2}, \]  

\[ X'(\pi) = m_1 m_2 (e^{\pi^2} - e^{\pi_1^2}), \]  

\[ m_{1,2} = \frac{Pe^* \pm \sqrt{Pe^* + 4\beta_n^* (L/R)^2}}{2} \]

and the eigenvalues are given by expression (14).

In this case, the correlation for \( Nu \) is obtained as

\[ Nu = \frac{\sum_{n=1}^{\infty} \frac{X(\pi)}{X(0) - X'(0)/Pe^*}}{\sum_{n=1}^{\infty} \frac{(X(\pi))/(X(0) - X'(0)/Pe^*)/\beta_n^*}. \]  

(19)

Both expressions Eqs. (16) and (19) are derived for constant wall temperature. The correlation can be used to determine the convective heat transfer coefficient between the nanofluid and the inner tube wall to estimate the enhanced heat transfer performance of the nanophase particles. Evidently, iteration is necessary. In a similar way, one can derive the \( Nu \) correlation corresponding to the other boundary conditions in the radial direction.

The enhanced performance of the nanofluid results from not only its high thermal conductivity, but also from the random movement and dispersion effect of the nanoparticles. The Peclet number \( Pe^* \) is a comprehensive parameter to describe such effects (as shown in Fig. 6). Experiment is necessary to determine this parameter. Compared to Choi’s primitive analysis (1995), which just accounted for the effect of high thermal conductivity of the nanofluid, the aforementioned expressions provide a sophisticated way to analyze the enhanced heat transfer mechanism of the nanofluid. It is emphasized that these expressions are theoretical and experimental work is needed to further reveal the enhanced mechanism and to improve the heat transfer performance of the nanofluid.

6. Conclusions

A preparation method of nanofluids has been developed. With this method, several sampled nanofluids have been prepared by directly mixing nanophase powders and base fluids, which reveals the possibility of practical application of the nanofluid. The nanofluid shows great potential in enhancing the heat transfer process. One reason is that the suspended ultra-fine particles remarkably increase the thermal conductivity of the nanofluid. The volume fraction, shape, dimensions and properties of the nanoparticles affect the thermal conductivity of nanofluids. The hot-wire method has been used to measure the thermal conductivity of nanofluids. The measurement results illustrate that the thermal conductivity of nanofluids remarkably increases with the volume fraction of ultra-fine particles. For the water–Cu nanoparticles suspension, for example, the ratio of the thermal conductivity of the nanofluid to that of the base liquid varies from 1.24 to 1.78 if the volume fraction of the ultra-fine particles increases from 2.5% to 7.5%.

With respect to the complicated phenomena of Brownian diffusion, sedimentation, dispersion which may coexist in the main flow of a nanofluid, the dispersion model has been used to analyze the enhanced heat transfer mechanism of nanofluid. Some correlations for predicting \( Nu \) have been derived. However, experimental research is urgently needed to investigate the heat transfer process of nanofluids.

References


